Module 2f – Normal Distributions

[Review Against All Odds: Unit 7]

[Review Against All Odds: Unit 8]

Normal Distribution
(Quantitative/Continuous variables)

- **Normal Distribution:** A special family of curves that tend to occur naturally and can usually be applied when analyzing a sample

- **1. It is symmetric**
- **2. It is unimodal**
- **3. It is bell-shaped**

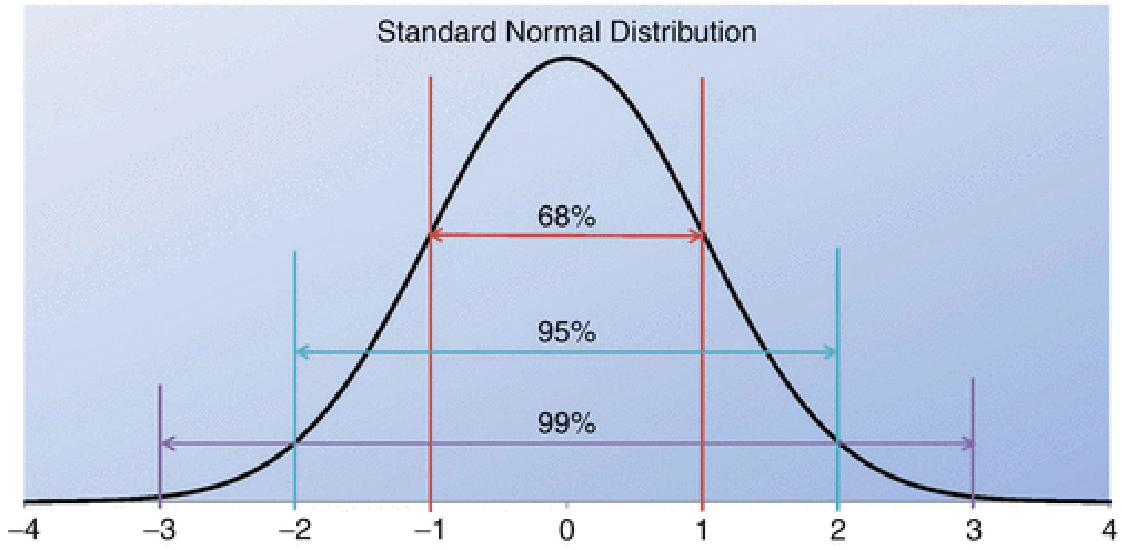
Normal Distribution

- A normal distribution is fully described by the mean and standard deviation value.

Normal Probability Density Function

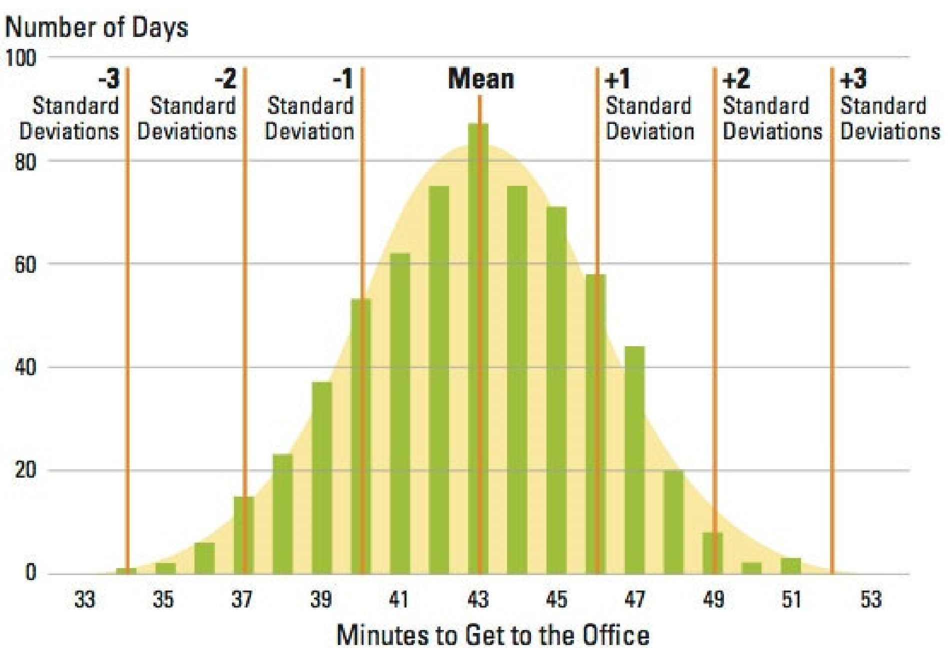
$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2}$$

- 68/95/99.7 Rule



Normal Distributions are Powerful

- Normal distributions occur "naturally":
  - Minutes to get to the office
  - Birthweight of babies
  - Scores on standardized tests



Normal Distributions are Powerful

- Normal distributions can be used in sampling
  - Based on the normal distribution, we can generally assume that sample results are a reasonably good estimation of the population.
  - This is the basis for the ***Central Limit Theorem*** (we'll talk about this more later)

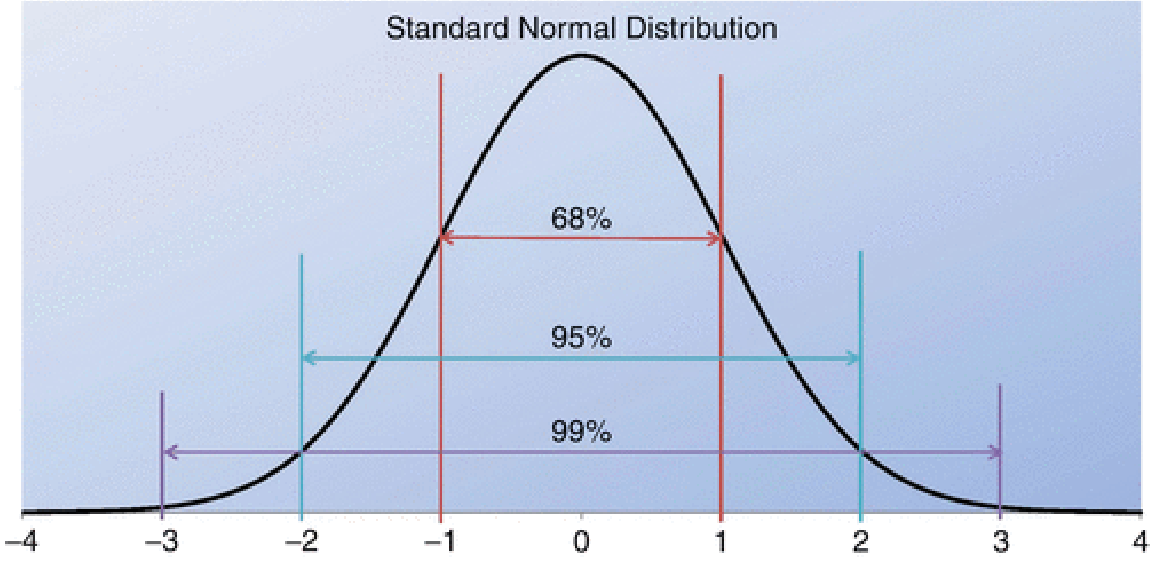
Density Curve

- Represents all individuals in a sample or population.
- Think of a histogram where all bars collectively represent all "individuals"
- The area under the curve adds up to 100% or 1.0

- Normal distributions are specific type of density curve defined entirely by mean and standard deviation.
  - Mean determines the center of the distribution on the x-axis
  - Standard defines how spread out or clustered together is the distribution

Example

- Suppose we know that IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. We can picture that normal distribution:



55 70 85 100 115 130 145

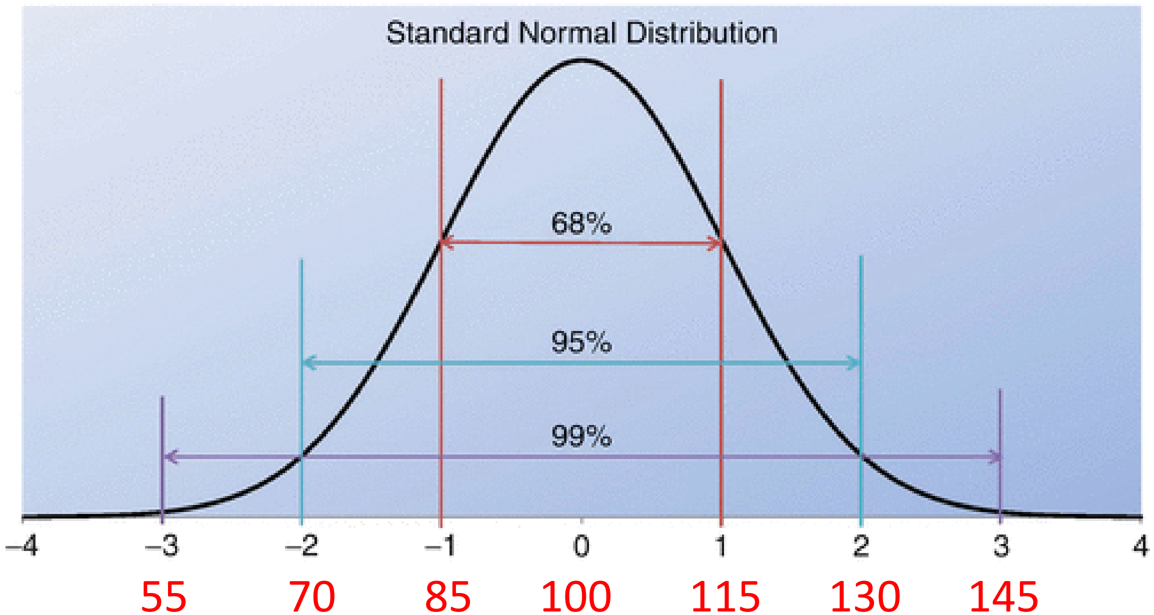
Example

- Standard Score/Z-Score

$$z = \frac{x - \mu}{\sigma}$$

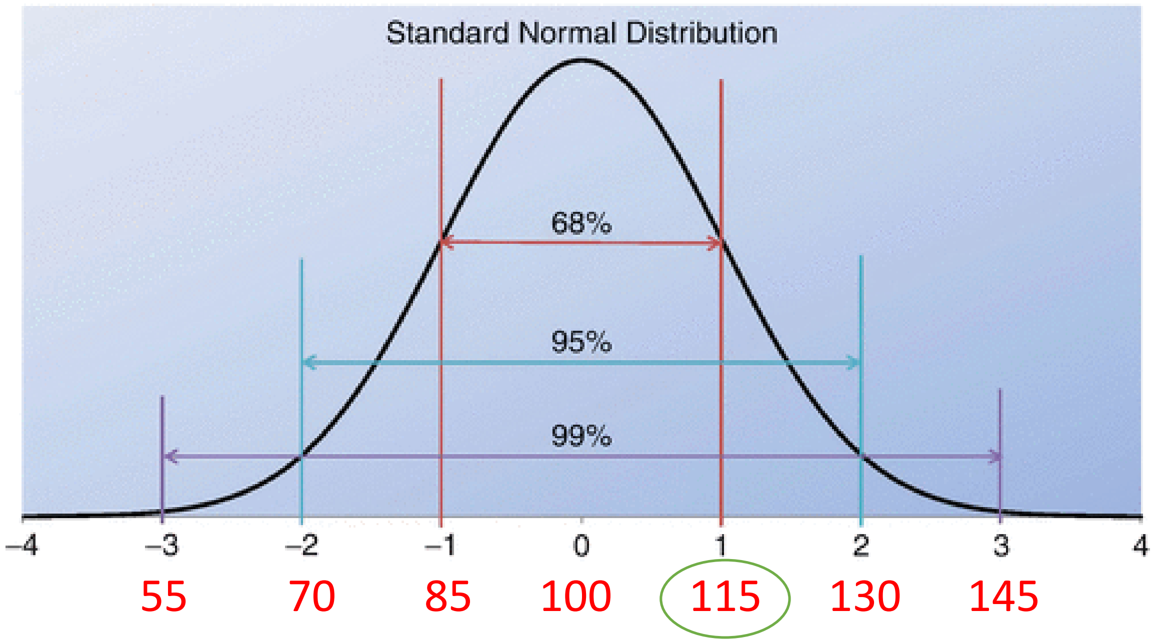
$\mu =$ Mean
$\sigma =$ Standard Deviation

- If Ana scores 115 on her IQ test, their z-score=(115-100)/15
- Her standard score/z-score is 1. The score is 1.0 standard deviations above the mean



Example

- Percentile: The percentile tells us the percentage of a distribution that is below a given value.
  - We can determine this mathematically using the diagram below.
  - We can also use Table A in our textbook



Example

- Ana’s percentile is 0.84 or 84%.
- She scored higher than about 84% of those who took the IQ test.

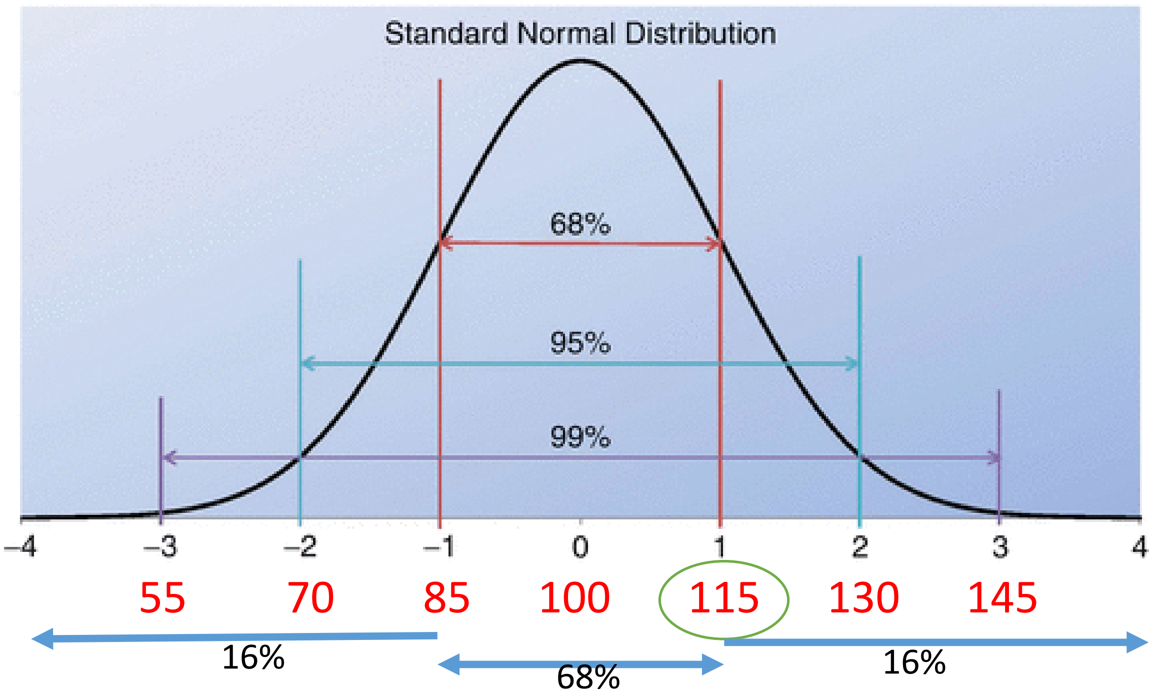


Table entry for z is the area under the standard Normal curve to the left of z.

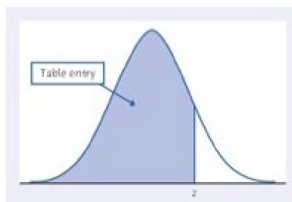


TABLE A Standard Normal cumulative proportions (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	[illegible]	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	[illegible]	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319