

# Module 10a – Inference for Means (continued)

[Review Against All Odds: Unit 27](#) (Inference)

# Stat Procedure Diagram – Where we are

|              |  | <u>Descriptive Statistics (Describing Pops or Samples)</u> |  | <u>Inferential Statistics (from Samples)</u>  |
|--------------|--|--|--|---|
|              | Variable Types                                 | Display  | Describe   | Estimation  |
| Univariate   | categorical (nominal or ordinal)*              | Bar Graph/Pie Chart  | Counts/Percentages                                       | When binary/dichotomous:<br>Confidence interval for proportions                     |
|              | quantitative/continuous                        | Histogram/Stem & Leaf<br>Box Plot                          | Mean/St Dev (normal)<br>Median/Min, Q1, Q3, Max (skewed) | Confidence interval for means   |
|              |  | Display  | Describe   | Significance Tests/Hypothesis Tests   |
| Bivariate    | 2 categorical                                  | Tables or Bar Graphs                                       | Two-way tables/Crosstabulation                           | Chi-square test (for goodness of fit)   |
|              | 1 categorical, 1 quant.                        | Bar Graphs   | Comparison of means/averages                             | T-test (one sample/group, two samples/groups)<br>ANOVA (two or more samples/groups) |
|              | 2 quant.                                       | Scatterplot  | Correlation Coef. (Coef. of determ)/<br>Regression Line  | T-test for correlation  |
|              |  | Display  | Describe   | Significance Tests/Hypothesis Tests   |
| Multivariate | Response Variable is Quant.                    | -  | Ordinary Least Squares Regression (OLS)                  | F-test for overall model<br>T-tests for each explanatory variable                   |
|              | Response Variable is categorical (dichotomous) | -  | Logistic Regression                                      | Chi-square tests of significance  |

NOTE: Items highlighted in yellow are covered in this course.

\*When a categorical variable has two categories, it is called dichotomous.

Still focused on...

...*quantitative/continuous variables (inference)*

- Confidence Intervals (with ***unknown*** population standard deviation)

$$\bar{x} \pm t \frac{s}{\sqrt{n}}$$

- Done exactly as before except we use our *degrees of freedom* to determine the t-value for the equation.
- Degrees of Freedom (df) for confidence intervals: **n-1**, where n=number of cases in our sample

# Confidence Intervals (practice)

Taken from Example 20.3 on page 459:

Calculating the average tip at a restaurant.

We randomly select 20 restaurant receipts out of all the receipts for an entire day.

Assume we know that the distribution of tips (as a percentage of the bill) is normally distributed but we don't know the population standard deviation so we substitute the **sample standard deviation**, which is given as **1.963%**.

We calculate the mean tip percentage for the **sample of 20** to be **22.21%**. Now we want to calculate the **95% confidence interval** for tips at the restaurant.

# Confidence Intervals (assumptions)

Calculating a confidence interval requires making three assumptions.

1. The data are independent observations from a ***simple random sample***.
2. The distribution of the underlying population is ***relatively normal***.
3. We do NOT know the population ***standard deviation*** (so we estimate it based on the sample standard deviation)

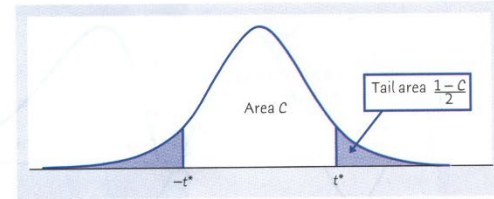
# t-distribution table (from page 701)

**Confidence Level: 95%**

**Degrees of Freedom  
(df): n-1 (20-1=19)**

**t=2.093**

Table entry for C is the critical value  $t^*$  required for confidence level C.  
To approximate one- and two-sided P-values, compare the value of the  $t$  statistic with the critical values of  $t^*$  that match the P-values given at the bottom of the table.



**TABLE C t DISTRIBUTION CRITICAL VALUES**

| DEGREES OF FREEDOM | CONFIDENCE LEVEL C |       |       |       |       |       |       |       |       |       |       |       |
|--------------------|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|                    | 50%                | 60%   | 70%   | 80%   | 90%   | 95%   | 96%   | 98%   | 99%   | 99.5% | 99.8% | 99.9% |
| 1                  | 1.000              | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2                  | 0.816              | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3                  | 0.765              | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4                  | 0.741              | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5                  | 0.727              | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6                  | 0.718              | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7                  | 0.711              | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8                  | 0.706              | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9                  | 0.703              | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10                 | 0.700              | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11                 | 0.697              | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12                 | 0.695              | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13                 | 0.694              | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14                 | 0.692              | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15                 | 0.691              | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16                 | 0.690              | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17                 | 0.689              | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18                 | 0.688              | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19                 | 0.688              | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20                 | 0.687              | 0.860 | 1.064 | 1.325 | 1.725 | 2.088 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21                 | 0.686              | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22                 | 0.686              | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23                 | 0.685              | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24                 | 0.685              | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25                 | 0.684              | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26                 | 0.684              | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27                 | 0.684              | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28                 | 0.683              | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29                 | 0.683              | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30                 | 0.683              | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40                 | 0.681              | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50                 | 0.679              | 0.849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60                 | 0.679              | 0.848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80                 | 0.678              | 0.846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100                | 0.677              | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000               | 0.675              | 0.842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| z*                 | 0.674              | 0.841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
| One-sided P        | .25                | .20   | .15   | .10   | .05   | .025  | .02   | .01   | .005  | .0025 | .001  | .0005 |
| Two-sided P        | .50                | .40   | .30   | .20   | .10   | .05   | .04   | .02   | .01   | .005  | .002  | .001  |

# Confidence Intervals (practice)

Mean: 22.21%

Sample Std Deviation: 1.963%

Number in our sample: 20

t-value: 2.093

$$\bar{X} \pm 2.093 * \frac{s}{\sqrt{n}}$$

$$22.21 \pm 2.093 * [1.963 / \sqrt{20}]$$

$$22.21 \pm .92$$

21.29 (LCL) to 23.13 (UCL)

We are 95% confident that the true mean percentage tip from all patrons at the restaurant is between 21.29% percent and 23.13%